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τ*-OPEN SETS AND RELATION BETWEEN SOME WEAK AND STRONG FORMS OF T*-OPEN SETS IN TOPOLOGICAL SPACES

Hariwan Zikri Ibrahim
Department of Mathematics, Faculty of Science, University of Zakho, Kurdistan-Region, Iraq

ABSTRACT
The main goal of this paper is to study some new classes of sets by using the open sets and functions in topological spaces. For this aim, the notions of τ*-open, τ-τ*-α-open, τ-τ*-preopen, τ-τ*-semiopen, τ-τ*-b-open, τ-τ*-β-open, τ*-τ-α-open, τ*-τ-preopen, τ*-τ-semiopen, τ*-τ-b-open, τ*-τ-β-open, τ*-α-open, τ*-preopen, τ*-semiopen, τ*-b-open and τ*-β-open sets are introduced. Properties and the relationships of τ*-open sets are investigated. Finally, the relationships between these sets and the related concepts are investigated.

KEYWORDS: open sets; τ*-open sets; functions.

Introduction
Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets. One of the most well known notions and also an inspiration source is the notion of *b*-open [2] sets introduced by Andrijevic in 1996, this class is a subclass of the class of β-open sets [1]. Also, the class of *b*-open sets is a superset of the class of semiopen sets [3] and the class of preopen sets [4]. In 1965, Njastad [5] defined the class of α-open sets.

Throughout this paper, *X* and *Y* refer always to topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset *A* of *X*, Cl(*A*) and Int(*A*) denote the closure of *A* and the interior of *A* in *X*, respectively.

Definition 1.1. A subset A of a space X is said to be:
1. α-open [5] if A ⊆ int(cl(int(A))).
2. preopen [4] if A ⊆ int(cl(A)).
3. semiopen [3] if A ⊆ cl(int(A)).
4. b-open [2] if A ⊆ int(cl(A)) ∪ cl(int(A)).
5. β-open [1] if A ⊆ cl(int(cl(A))).

τ*-OPEN SETS
Definition 2.1. Let (X, τ) and (Y, δ) be two topological spaces and let f be a function from X into Y, then a subset A in τ is called τ*-open if there exists an open set G in δ such that $A = f^{-1}(G)$, that is $\tau^* = \{f^{-1}(G): G \in \delta \text{ and } f^{-1}(G) \in \tau\}$. The family of all τ*-open sets in X is denoted by τ*.

Remark 2.2. From the definition of function it is clear that $f^{-1}(\emptyset) = \emptyset \in \tau$ and $f^{-1}(Y) = X \in \tau$, that is ∅ ∈ τ* and X ∈ τ* for any function f from a space X into a space Y.

Remark 2.3. It is clear that from the definition that every τ*-open subset of a space X is open in X, but the converse is not true in general as shown in the following example.

Example 2.4. Consider X = {a, b, c} with the topology τ = {∅, X, {b}, {a, b}, {b, c}} and Y = {1, 2, 3} with the topology δ = {∅, Y, {1}, {1, 2}}. Define a function f: X ⟶ Y as follows f(a) = 2, and f(b) = f(c) = 1. Then τ* = {∅, X, {b, c}} and {b} ∈ τ, but {b} ∉ τ*.

Theorem 2.5. Let f: X ⟶ Y be a function and let $\{A_\alpha\}_{\alpha\in J}$ be a collection of τ*-open sets, then $\cup_{\alpha\in J} A_\alpha$ is τ*-open.

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Proof. Let A_α be a τ*-open set for each α, then A_α is open in X and hence $\cup_{\alpha\in J} A_\alpha$ is open. Since for each A_α there exist $G_\alpha \in \delta$ such that $A_\alpha = f^{-1}(G_\alpha) \in \tau$ and $\cup_{\alpha\in J} f^{-1}(G_\alpha) \in \tau$. Now, $\cup_{\alpha\in J} A_\alpha = \cup_{\alpha\in J} f^{-1}(G_\alpha) = f^{-1}(\cup_{\alpha\in J} G_\alpha)$ and $\cup_{\alpha\in J} G_\alpha \in \delta$. Therefore, $\cup_{\alpha\in J} A_\alpha$ is a τ*-open set.

Theorem 2.6. Let f: X ⟶ Y be any function. If A and B are τ*-open sets, then A ∩ B is also a τ*-open set.

Proof. Let $A = f^{-1}(G_1)$ and $B = f^{-1}(G_2)$ be two τ*-open sets for some $G_1, G_2 \in \delta$ and hence A and B are open in X, then $G_1 \cap G_2 \in \delta$, $A \cap B \in \tau$ and $A \cap B = f^{-1}(G_1) \cap f^{-1}(G_2) = f^{-1}(G_1 \cap G_2)$. Therefore, A ∩ B is a τ*-open set.

Remark 2.7. From Remark 2.2, Theorems 2.5 and 2.6, we notice that the family of all τ*-open subsets of a space X is a topology on X.

Proposition 2.8. The set A is τ*-open in the space X if and only if for each x ∈ A, there exists a τ*-open set B such that x ∈ B ⊆ A.

Proof. Assume that A is τ*-open set in X, then for each x ∈ A, put A = B is τ*-open set containing x such that x ∈ B ⊆ A.
Conversely, suppose that for each x ∈ A, there exists a τ*-open set B such that x ∈ B ⊆ A, thus $A = \cup B_x$ where $B_x \in \tau^*$ for each x, therefore A is τ*-open set.

Definition 2.9. A subset F of X is called τ*-closed if X \ F is τ*-open. The family of all τ*-closed sets in X is denoted by τ^{*c}.

Proposition 2.10. Let $\{F_j\}_{j\in\beta}$ be a collection of τ*-closed subsets of X. Then $\cap_{j\in\beta} F_j$ is τ*-closed.

Proof. Follows from Theorem 2.5.

Proposition 2.11. If A and B are τ*-closed sets in X, then A ∪ B is also a τ*-closed set.

Proof. Follows from Theorem 2.6.

Definition 2.12. Let f: X ⟶ Y be any function and let A be a subset of a topological space (X, τ).

1. The union of all τ*-open sets contained in A is called the τ*-interior of A and denoted by τ*-int(A).
2. The intersection of all τ*-closed sets containing A is called the τ*-closure of A and denoted by τ*-cl(A).

We now state the following theorem without proof.

Theorem 2.13. Let f: X ⟶ Y be any function. For any subsets A, B of X we have the following:

1. A is τ*-open if and only if A = τ*-int(A).
2. A is τ*-closed if and only if A = τ*-cl(A).
3. If A ⊆ B then τ*-int(A) ⊆ τ*-int(B) and τ*-cl(A) ⊆ τ*-cl(B).
4. τ*-int(A) ∪ τ*-int(B) ⊆ τ*-int(A∪B).
5. τ*-int(A ∩ B) = τ*-int(A) ∩ τ*-int(B) .
6. τ*-cl(A ∪ B) = τ*-cl(A) ∪ τ*-cl(B).
7. τ*-cl(A ∩ B) ⊆ τ*-cl(A) ∩ τ*-cl(B).
8. τ*-int(X \ A) = X \ τ*-cl(A).
9. τ*-cl(X \ A) = X \ τ*-int(A).

Theorem 2.14. Let f: X ⟶ Y be a function and let A be a subset of X. Then x ∈ τ*-cl(A) if and only if for every τ*-open set V of X containing x, A ∩ V ≠ ∅.

Proof. Let x ∈ τ*-cl(A) and suppose that A ∩ V = ∅ for some τ*-open set V which contains x. Then X \ V is τ*-closed and A ⊆ X\V, thus τ*-cl(A) ⊆ X \ V. But this implies that x ∈ X\V, which is contradiction. Therefore A ∩ V ≠ ∅.
Conversely, let A ⊆ X and x ∈ X such that for each τ*-open set V of X contains x, A ∩ V ≠ ∅. If x ∉ τ*-cl(A), then there is a τ*-closed set F such that A ⊆ F and x ∉ F. Hence X \ F is a τ*-open set with x ∈ X \ F and thus X \ F ∩ A ≠ ∅, which is contradiction. Therefore x ∈ τ*-cl(A).

Proposition 2.15. Let f: X ⟶ Y be a function. Then:

1. For every τ*-open set G and for every subset A ⊆ X we have τ*-cl(A) ∩ G ⊆ τ*-cl(A ∩ G).
2. For every τ*-closed set F and for every subset A ⊆ X we have τ*-int(A ∪ F) ⊆ τ*-int(A) ∪ F.

Proof.

1. Let x ∈ τ*-cl(A) ∩ G, then x ∈ τ*-cl(A) and x ∈ G. Let V be the τ*-open set containing x. Then by Theorem 2.6, V ∩ G is also τ*-open set containing x. Since x ∈ τ*-cl(A) implies (V ∩ G) ∩ A ≠ ∅. This implies V ∩ (G ∩ A) ≠ ∅. This is true for every V containing x, hence by Theorem 2.14, x ∈ τ*-cl(G ∩ A). Therefore τ*-cl(A) ∩ G ⊆ τ*-cl(A ∩ G).
2. Follows from (1) and Theorem 2.13.

Corollary 2.16. Let A be any subset of X. Then the following relation holds,

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τ^*-int(A) ⊆ int(A) ⊆ A ⊆ cl(A) ⊆ τ^*-cl(A).

Proof. Follows from Remark 2.3.

Definition 2.17. Let f: X ⟶ Y be any function. A subset A of a space X is said to be:

1. τ-τ^*-α-open if A ⊆ int(τ^*-cl(int(A))).
2. τ-τ^*-preopen if A ⊆ int(τ^*-cl(A)).
3. τ-τ^*-semiopen if A ⊆ τ^*-cl(int(A)).
4. τ-τ^*-b-open if A ⊆ int(τ^*-cl(A)) U τ^*-cl(int(A)).
5. τ-τ^*-β-open if A ⊆ τ^*-cl(int(τ^*-cl(A))).
6. τ^*-τ-α-open if A ⊆ τ^*-int(cl(τ^*-int(A))).
7. τ^*-τ-preopen if A ⊆ τ^*-int(cl(A)).
8. τ^*-τ-semiopen if A ⊆ cl(τ^*-int(A)).
9. τ^*-τ-b-open if A ⊆ τ^*-int(cl(A)) ∪ cl(τ^*-int(A)).
10. τ^*-τ-β-open if A ⊆ cl(τ^*-int(cl(A))).
11. τ^*-α-open if A ⊆ τ^*-int(τ^*-cl(τ^*-int(A))).
12. τ^*-preopen if A ⊆ τ^*-int(τ^*-cl(A)).
13. τ^*-semiopen if A ⊆ τ^*-cl(τ^*-int(A)).
14. τ^*-b-open if A ⊆ τ^*-int(τ^*-cl(A)) ∪ τ^*-cl(τ^*-int(A)).
15. τ^*-β-open if A ⊆ τ^*-cl(τ^*-int(τ^*-cl(A))).

Proposition 2.18. Let f: X ⟶ Y be any function, then the following statements are hold:

1. Every open set is τ-τ^*-α-open.
2. Every τ-τ^*-α-open set is τ-τ^*-preopen.
3. Every τ-τ^*-α-open set is τ-τ^*-semiopen.
4. Every τ-τ^*-preopen set is τ-τ^*-b-open.
5. Every τ-τ^*-semiopen set is τ-τ^*-b-open.
6. Every τ-τ^*-b-open set is τ-τ^*-β-open.
7. The concepts of τ-τ^*-preopen and τ-τ^*-semiopen are independent.

Proof. Obvious.

Remark 2.19. The converse of the above proposition need not be true as shown by the following examples.

Example 2.20. Consider X = {a, b, c} with τ = {∅, X, {a}, {a, c}} and Y = {1, 2, 3} with δ = {∅, Y, {1}}. Let A = {a, b} and f: X ⟶ Y be a function such that f(a) =1, f(b) = 2 and f(c) = 3. Then τ^*={∅, X, {a}} and A is τ-τ^*-α-open but A is not open.

Example 2.21. Consider X = {a, b, c} with τ = {∅, X, {b}, {a, b}, {b, c}} and Y = {1, 2, 3} with δ = {∅, Y, {1}, {1, 2}}. Let A = {a, c} and f: X ⟶ Y be a function such that f(a) = 2 and f(b) = f(c) = 1. Then τ^* = {∅, X, {b, c}} and A is τ-τ^*-preopen but A is not τ-τ^*-α-open. Also A is τ-τ^*-b-open but A is not τ-τ^*-semiopen.

Example 2.22. Consider X = {a, b, c} with τ = {∅, X, {b}, {c}, {b, c}} and Y = {1, 2, 3} with δ = {∅, Y, {2}, {3}, {2, 3}}. Let A = {a, b} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = 2 and f(c) = 3. Then τ^*={∅, X, {b}, {c}, {b, c}} and A is τ-τ^*-semiopen but A is not τ-τ^*-α-open. Also A is τ-τ^*-b-open but A is not τ-τ^*-preopen.

Example 2.23. Consider X = {1, 2, 3, 4, 5} with τ = {∅, X, {1}, {1, 2}, {3, 4, 5}, {1, 3, 4, 5}} and Y = {a, b, c} with δ = {∅, Y, {a}}. Let A = {2, 3} and f: X ⟶ Y be a function such that f(1) = a, f(2) = b and f(3) = f(4) = f(5) = c. Then τ^*={∅, X, {1}} and A is τ-τ^*-β-open but A is not τ-τ^*-b-open.

Proposition 2.24. Let f: X ⟶ Y be any function, then the following statements are hold:

1. Every τ^*-open set is τ^*-τ-α-open.
2. Every τ^*-τ-α-open set is τ^*-τ-preopen.
3. Every τ^*-τ-α-open set is τ^*-τ-semiopen.
4. Every τ^*-τ-preopen set is τ^*-τ-b-open.
5. Every τ^*-τ-semiopen set is τ^*-τ-b-open.
6. Every τ^*-τ-b-open set is τ^*-τ-β-open.
7. The concepts of τ^*-τ-preopen and τ^*-τ-semiopen are independent.

Proof. Obvious.

Remark 2.25. The converse of the above proposition need not be true as shown by the following examples.

Example 2.26. Consider X = {a, b, c} with τ = {∅, X, {a}, {a, c}} and Y = {1, 2, 3} with δ = {∅, Y, {1}}. Let A = {a, b} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = 2 and f(c) = 3. Then τ^* ={∅, X, {a}} and A is τ^*-τ-α-open but A is not τ^*-open.

Example 2.27. Consider X = {a, b, c} with τ = {∅, X, {c}, {a, b}} and Y = {1, 2, 3} with δ = {∅, Y, {3}, {1, 2}}. Let A = {a} and f: X ⟶ Y be a function such that f(a) = f(b) = 2 and f(c) = 3. Then τ^* = {∅, X, {c}, {a, b}} and A is τ^*-τ- preopen but A is not τ^*-τ-α-open. Also A is τ^*-τ-b-open but A is not τ^*-τ-semiopen.

Example 2.28. Consider X = {a, b, c} with τ = {∅, X, {b}, {c}, {b, c}} and Y = {1, 2, 3} with δ = {∅, Y, {2}, {3}, {2, 3}}. Let A={a, b} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = 2 and f(c) = 3. Then τ^* ={∅, X, {b}, {c}, {b, c}} and A is τ^*-τ- semiopen

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but A is not τ*-τ-α-open. Also A is τ*-τ-b-open but A is not τ*-τ-preopen.

Example 2.29. Consider X = Y = {1, 2, 3, 4, 5} with τ = {∅, X, {1}, {2}, {1, 2}, {3, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}, {1, 3, 4, 5}} and δ = {∅, Y, {2}, {3, 4}, {2, 3, 4}, {1, 3, 4, 5}}. Let A= {2, 3, 5} and f: X ⟶ Y be a function such that f(1) = 1, f(2) = 2, f(3) = 3, f(4) = 4 and f(5) = 5. Then τ* = {∅, X, {2}, {3, 4}, {2, 3, 4}, {1, 3, 4, 5}} and A is τ*-τ-β-open but A is not τ*-τ-b-open.

Proposition 2.30. Let f: X ⟶ Y be any function, then the following statements are hold:

1. Every τ*-open set is τ*-α-open.
2. Every τ*-α-open set is τ*-preopen.
3. Every τ*-α-open set is τ*-semiopen.
4. Every τ*-preopen set is τ*-b-open.
5. Every τ*-semiopen set is τ*-b-open.
6. Every τ*-b-open set is τ*-β-open.
7. The concepts of τ*-preopen and τ*-semiopen are independent.

Proof. Obvious.

Remark 2.31. The converse of the above proposition need not be true as shown by the following examples.

Example 2.32. Consider X = {a, b, c} with τ = {∅, X, {a}, {a, c}} and Y = {1, 2, 3} with δ = {∅, Y, {1}}. Let A = {a, b} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = 2 and f(c) = 3. Then τ* ={∅, X, {a}} and A is τ*-α-open but A is not τ*-open.

Example 2.33. Consider X = {a, b, c} with τ = {∅, X, {c}, {a, b}} and Y = {1, 2, 3} with δ = {∅, Y, {3}, {1, 2}}. Let A = {a} and f: X ⟶ Y be a function such that f(a) = f(b) = 2 and f(c) = 3. Then τ* = {∅, X, {c}, {a, b}} and A is τ*-preopen but A is not τ*-α-open. Also A is τ*-b-open but A is not τ*-semiopen.

Example 2.34. Consider X = {a, b, c} with τ ={∅, X, {b}, {c}, {b, c}} and Y={1, 2, 3} with δ = {∅, Y, {2}, {3}, {2, 3}}. Let A = {a, b} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = 2 and f(c) = 3. Then τ* ={∅, X, {b}, {c}, {b, c}} and A is τ*-semiopen but A is not τ*-α-open. Also A is τ*-b-open but A is not τ^*-preopen.

Example 2.35. Consider X = {1, 2, 3, 4, 5} with τ ={∅, X, {1}, {2}, {1, 2}, {3, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}, {1, 3, 4, 5}} and Y = {a, b, c, d, e} with δ = {∅, Y, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}, {a, b, c, d}, {a, c, d, e}}. Let A= {2, 3, 5} and f: X ⟶ Y be a function such that f(1) = a, f(2) = b, f(3) = c, f(4) = d and f(5) = e. Then τ* ={∅, X, {1}, {2}, {1, 2}, {3, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}, {1, 3, 4, 5}} and A is τ*-β-open but A is not τ*-b-open.

Proposition 2.36. Let f: X ⟶ Y be any function, then the following statements are hold:

1. Every τ*-τ-α-open set is τ-τ*-α-open.
2. Every τ*-τ-preopen set is τ-τ*-preopen.
3. Every τ*-τ-semiopen set is τ-τ*-semiopen.
4. Every τ*-τ-b-open set is τ-τ*-b-open.
5. Every τ*-τ-β-open set is τ-τ*-β-open.

Proof. Follows from Corollary 2.16.

Remark 2.37. The converse of the above proposition need not be true as shown by the following example.

Example 2.38. Consider X = {a, b, c} with τ = {∅, X, {a}, {b}, {a, b}} and Y= {1, 2, 3} with δ = {∅, Y, {1}}. Let A= {a} and f: X ⟶ Y be a function such that f(a) = 3, f(b) = 1 and f(c) = 2. Then τ* ={∅, X, {b}} and A is τ-τ*-α-open, τ-τ*-preopen, τ-τ*-semiopen, τ-τ*-b-open and τ-τ*-β-open but A is not τ*-τ-α-open, τ*-τ-preopen, τ*-τ-semiopen, τ*-τ-b-open and τ*-τ-β-open.

Proposition 2.39. Let f: X ⟶ Y be any function, then the following statements are hold:

1. Every τ*-τ-α-open set is τ*-α-open.
2. Every τ*-τ-preopen set is τ*-preopen.
3. Every τ*-τ-semiopen set is τ*-semiopen.
4. Every τ*-τ-b-open set is τ*-b-open.
5. Every τ*-τ-β-open set is τ*-β-open.

Proof. Follows from Corollary 2.16.

Remark 2.40. The converse of the above proposition need not be true as shown by the following examples.

Example 2.41. Consider X = {a, b, c} with τ = {∅, X, {a}, {b, c}} and Y= {1, 2, 3} with δ = {∅, Y}. Let A= {a} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = 2, f(c) = 3. Then τ*= {∅, X} and A is τ*-preopen, τ*-b-open and τ*-β-open but A is not τ*-τ-preopen, τ*-τ-b-open and τ*-τ-β-open.

Example 2.42. Consider X = {a, b, c} with τ = {∅, X, {a}, {b}, {a, b}} and Y= {1, 2, 3} with δ = {∅, Y,

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{1}}. Let A= {a, b} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = f(c) = 3. Then τ* = {∅, X, {a}} and A is both τ*-α-open and τ*-semiopen but A is neither τ*-τ-α-open nor τ*-τ-semiopen.

Proposition 2.43. Let f: X ⟶ Y be any function, then the following statements are hold:
1. Every α-open set is τ-τ*-α-open.
2. Every preopen set is τ-τ*-preopen.
3. Every semiopen set is τ-τ*-semiopen.
4. Every b-open set is τ-τ*-b-open.
5. Every β-open set is τ-τ*-β-open.

Proof. Follows from Corollary 2.16.

Remark 2.44. The converse of the above proposition need not be true as shown by the following examples.

Example 2.45. Consider X = {a, b, c} with τ = {∅, X, {a}, {b, c}} and Y = {1, 2, 3} with δ = {∅, Y}. Let A = {a, b} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = 2 and f(c) = 3. Then τ* = {∅, X} and A is τ-τ*-α-open but A is not α-open. Also A is τ- τ*-semiopen but A is not semiopen.

Example 2.46. Consider X = {a, b, c} with τ = {∅, X, {b}, {a, b}, {b, c}} and Y = {1, 2, 3} with δ = {∅, Y, {1}, {1, 2}}. Let A = {a, c} and f: X ⟶ Y be a function such that f(a) = 2 and f(b) = f(c) = 1. Then τ* = {∅, X, {b, c}} and A is τ-τ*-preopen but A is not preopen. Also A is τ-τ*-b-open but A is not b-open. Also A is τ-τ*-β-open but A is not β-open.

Proposition 2.47. Let f: X ⟶ Y be any function, then the following statements are hold:
1. Every τ^*-τ-α-open set is α-open.
2. Every τ^*-τ-preopen set is preopen.
3. Every τ^*-τ-semiopen set is semiopen.
4. Every τ^*-τ-b-open set is b-open.
5. Every τ^*-τ-β-open set is β-open.

Proof. Follows from Corollary 2.16.

Remark 2.48. The converse of the above proposition need not be true as shown by the following example.

Example 2.49. Consider X = {a, b, c} with τ = {∅, X, {a}, {b}, {a, b}} and Y = {1, 2, 3} with with δ = {∅, Y, {3}}. Let A = {a} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = 3, and f(c) = 2. Then τ* = {∅, X, {b}} and A is α-open, preopen, semiopen, b-open and β-open but A is not τ*-τ-α-open, τ*-τ-preopen, τ*-τ-semiopen, τ*-τ-b-open and τ*-τ-β-open.

Proposition 2.50. Let f: X ⟶ Y be any function, then the following statements are hold:
1. Every τ*-α-open set is τ-τ*-α-open.
2. Every τ*-preopen set is τ-τ*-preopen.
3. Every τ*-semiopen set is τ-τ*-semiopen.
4. Every τ*-b-open set is τ-τ*-b-open.
5. Every τ*-β-open set is τ-τ*-β-open.

Proof. Follows from Corollary 2.16.

Remark 2.51. The converse of the above proposition need not be true as shown by the following example.

Example 2.52. Consider X = {a, b, c} with τ = {∅, X, {a}, {b}, {a, b}} and Y = {1, 2, 3} with with δ = {∅, Y, {2}}. Let A = {a} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = 2, and f(c) = 3. Then τ* = {∅, X, {b}} and A is τ-τ*-α-open, τ-τ*-preopen, τ-τ*-semiopen, τ-τ*-b-open and τ-τ*-β-open but A is not τ*-α-open, τ*-preopen, τ*-semiopen, τ*-b-open and τ*-β-open.

Proposition 2.53. Let f: X ⟶ Y be any function, then the following statements are hold:
1. The concepts of τ*-α-open and α-open are independent.
2. The concepts of τ*-preopen and preopen are independent.
3. The concepts of τ*-semiopen and semiopen are independent.
4. The concepts of τ*-b-open and b-open are independent.
5. The concepts of τ*-β-open and β-open are independent.

Remark 2.54. The converse of the above proposition need not be true as shown by the following examples.

Example 2.55. Consider X = {a, b, c} with τ = {∅, X, {a}, {b}, {a, b}} and Y = {1, 2, 3} with δ = {∅, Y, {3}}. Let A= {a} and f: X ⟶ Y be a function such that f(a) = 2, f(b) = 3 and f(c) = 1. Then τ* = {∅, X, {b}} and A is α-open, preopen, semiopen, b-open and β-open but A is not τ*-α-open, τ*-preopen, τ*-semiopen, τ*-b-open and τ*-β-open.

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Example 2.56. Consider X = {a, b, c} with τ = {∅, X, {b}, {a, b}, {b, c}} and Y = {1, 2, 3} with δ = {∅, Y, {1, 2}}. Let A= {a, c} and f: X ⟶ Y be a function such that f(a) = 3, f(b) = 1 and f(c) = 2. Then τ* = {∅, X, {b, c}} and A is τ*-preopen, τ*-b-open and τ*-β-open but A is not preopen, b-open and β-open.

Example 2.57. Consider X = {a, b, c} with τ = {∅, X, {b}, {c}, {b, c}} and Y = {1, 2, 3} with δ = {∅, Y, {2}}. Let A= {a, c} and f: X ⟶ Y be a function such that f(a) = 1, f(b) = 3 and f(c) = 2. Then τ* = {∅, X, {c}} and A is τ*-α-open but A is not α-open.

Example 2.58. Consider X = {a, b, c, d} with τ = {∅, X, {c}, {a, b}, {a, b, c}} and Y = {1, 2, 3} with δ = {∅, Y, {1}}. Let A= {a, c} and f: X ⟶ Y be a function such that f(a) = f(d) = 2, f(b) = 3 and f(c) = 1. Then τ* = {∅, X, {c}} and A is τ*-semiopen but A is not semiopen.

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